S2(R) S14

1. Before Roger will use a tennis ball he checks it using a "bounce" test. The probability that a ball from Roger's usual supplier fails the bounce test is 0.2. A new supplier claims that the probability of one of their balls failing the bounce test is less than 0.2. Roger checks a random sample of 40 balls from the new supplier and finds that 3 balls fail the bounce test.

Stating your hypotheses clearly, use a 5% level of significance to test the new supplier's claim.

(5)

x = # balls that fail bounce test

$x \sim B(40, 0.2)$

$H_0 : p = 0.2$ $\quad P(x \leq 3) = 0.0285$

$H_1 : p < 0.2$

$< 5\%$ $\therefore$ result is significant

$\therefore$ reject null hypothesis

$\therefore$ evidence to support new supplier's claim

2. A bag contains a large number of counters. Each counter has a single digit number on it and the mean of all the numbers in the bag is the unknown parameter μ. The number 2 is on 40% of the counters and the number 5 is on 25% of the counters. All the remaining counters have numbers greater than 5 on them.

A random sample of 10 counters is taken from the bag.

(a) State whether or not each of the following is a statistic

(i) S = the sum of the numbers on the counters in the sample,

(ii) D = the difference between the highest number in the sample and μ,

(iii) F = the number of counters in the sample with a number 5 on them.

(3)

The random variable T represents the number of counters in a random sample of 10 with the number 2 on them.

(b) Specify the sampling distribution of T.

(2)

The counters are selected one by one.

(c) Find the probability that the third counter selected is the first counter with the number 2 on it.

(2)

a) x_i = number on counter i

i) S is a statistic $S = x_1 + .. + x_{10}$
this contains no unknown parameters

ii) D is not a statistic as μ is unknown

iii) F is a statistic, contains no unknown parame

b)

T	0	1	2	3	4	5	6	7	8	9	10
P											

$T \sim B(10, 0.4)$

c) $P(2'2'2) = 0.6 \times 0.6 \times 0.4 = 0.144$

3. Accidents occur randomly at a road junction at a rate of 18 every year.
The random variable X represents the number of accidents at this road junction in the next 6 months.

(a) Write down the distribution of X.

(2)

(b) Find $P(X > 7)$.

(2)

(c) Show that the probability of at least one accident in a randomly selected month is 0.777 (correct to 3 decimal places).

(3)

(d) Find the probability that there is at least one accident in exactly 4 of the next 6 months.

(3)

a) x = # accidents per 6 months $x \sim Po(9)$

b) $P(x > 7) = 1 - P(x \leq 7) = 0.6761$

c) y = # accidents per month $y \sim Po(1.5)$

$P(y \geq 1) = 1 - P(y = 0) = 1 - e^{-1.5} = 0.777(35+)$

d) b = # months with at least one accident

$b \sim B(6, 0.777)$

$P(b = 4) = {}^6C_4\, 0.777^4\, 0.223^2 = 0.272$

4. The random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} 3k & 0 \leqslant x < 1 \\ kx(4-x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Sketch $\mathrm{f}(x)$.

(3)

(b) Write down the mode of X.

(1)

Given that $\mathrm{E}(X) = \dfrac{29}{16}$

(c) describe, giving a reason, the skewness of the distribution.

(2)

(d) Use integration to find the value of k.

(5)

(e) Write down the lower quartile of X.

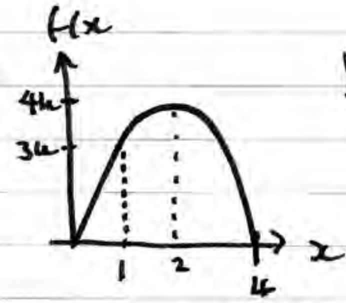
(1)

Given also that $\mathrm{P}(2 < X < 3) = \dfrac{11}{36}$

(f) find the exact value of $\mathrm{P}(X > 3)$.

(2)

a)



b) mode = 2

c) mean < mode

∴ negative skew

d) $\int f(x)dx = 1$

$= \int_0^1 3k\,dx + k\int_1^4 4x - x^2\,dx = [3kx]_0^1 + k[2x^2 - \frac{1}{3}x^3]_1^4 = 1$

$\therefore 3k + k(\frac{32}{3} - \frac{5}{3}) = 3k + 9k \Rightarrow 12k = 1 \quad \therefore k = \frac{1}{12}$

e) $F(1) = \int_0^1 f(x)dx = \int_0^1 \frac{3}{12}dx = [\frac{1}{4}x]_0^1 = \frac{1}{4} \quad F(Q_1) = \frac{1}{4}$

$\therefore Q_1 = 1$

f) $\frac{1}{12}\int_3^4 4x - x^2\,dx = \frac{1}{12}[2x^2 - \frac{1}{3}x^3]_3^4 = \frac{1}{12}(\frac{32}{3} - 9) = \frac{5}{36}$

5. Sammy manufactures wallpaper. She knows that defects occur randomly in the manufacturing process at a rate of 1 every 8 metres. Once a week the machinery is cleaned and reset. Sammy then takes a random sample of 40 metres of wallpaper from the next batch produced to test if there has been any change in the rate of defects.

(a) Stating your hypotheses clearly and using a 10% level of significance, find the critical region for this test. You should choose your critical region so that the probability of rejection is less than 0.05 in each tail.

(4)

(b) State the actual significance level of this test.

(2)

Thomas claims that his new machine would reduce the rate of defects and invites Sammy to test it. Sammy takes a random sample of 200 metres of wallpaper produced on Thomas' machine and finds 19 defects.

(c) Using a suitable approximation, test Thomas' claim. You should use a 5% level of significance and state your hypotheses clearly.

(7)

a) x = # defects per 40m $\quad x \sim Po(5)$

$H_0: \lambda = 5 \quad P(x \leq L) < 0.05 \quad P(x \geq u) < 0.05$

$H_1: \lambda \neq 5 \quad P(x \leq 1) = 0.0404 \quad P(x > u-1) < 0.05$

$P(x \leq 2) = 0.1247 \quad P(x \leq u-1) > 0.95$

$* \; L = 1 \quad P(x \leq 8) = 0.9319$

$P(x \leq 9) = 0.9682 \; *$

CR $\{x \leq 1\} \cup \{x \geq 10\}$ $\quad u-1 = 9 \; \therefore u = 10$

b) ASL = $\begin{array}{r} 0.0404 \\ 0.0318 \end{array} +$ $\quad 0.0722 \quad 7.22\%$

2

c) y = # defects per 200m $\quad y \sim Po(25)$

$\cancel{x}$ $y \sim N(25, 25)$

$P(y \leq 19)$ $\approx P(y \leq 19.5)$

$P(y < 20)$

$H_0: \lambda = 25$

$H_1: \lambda < 25$

$\approx P\left(z \leq \frac{19.5 - 25}{5}\right) = P(z \leq -1.1)$

$= 0.1357$

2

$> 5\%$ $\therefore$ not significant $\therefore$ cannot reject null

$\therefore$ not enough evidence to support Thomas' claim

6. In an experiment some children were asked to estimate the position of the centre of a circle. The random variable D represents the distance, in centimetres, between the child's estimate and the actual position of the centre of the circle. The cumulative distribution function of D is given by

$$\mathrm{F}(d) = \begin{cases} 0 & d < 0 \\ \dfrac{d^2}{2} - \dfrac{d^4}{16} & 0 \leqslant d \leqslant 2 \\ 1 & d > 2 \end{cases}$$

(a) Find the median of D.

(4)

(b) Find the mode of D.

Justify your answer.

(5)

The experiment is conducted on 80 children.

(c) Find the expected number of children whose estimate is less than 1 cm from the actual centre of the circle.

(3)

a) $F(\text{median}) = \frac{1}{2}$ $\quad \frac{d^2}{2} - \frac{d^4}{16} = \frac{1}{2}$

$\frac{d^4}{16} - \frac{d^2}{2} + \frac{1}{2} = 0$ $\quad (\times 16)$ $\quad d^4 - 8d^2 + 8 = 0$

$d^2 = \frac{8 \pm \sqrt{8^2 - 4(1)(8)}}{2}$ $\quad \therefore d = 1.08$

b) $f(d) = \frac{d}{dx}\left(\frac{d^2}{2} - \frac{d^4}{16}\right) = d - \frac{1}{4}d^3$

$f'(d) = 0$ when $d =$ mode $\quad f'(d) = 1 - \frac{3}{4}d^2 = 0$

$d^2 = \frac{4}{3}$ $\quad \therefore d = 1.155$ = mode.

c) $P(d < 1) = F(1) = \frac{1}{2} - \frac{1}{16} = \frac{7}{16}$

$(\times 80)$ $= 35$ children

7. A piece of string AB has length 9 cm. The string is cut at random at a point P and the random variable X represents the length of the piece of string AP.

(a) Write down the distribution of X.

(1)

(b) Find the probability that the length of the piece of string AP is more than 6 cm.

(1)

The two pieces of string AP and PB are used to form two sides of a rectangle.

The random variable R represents the area of the rectangle.

(c) Show that $R = aX^2 + bX$ and state the values of the constants a and b.

(2)

(d) Find E(R).

(6)

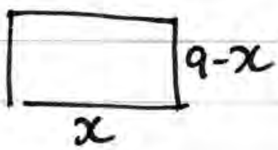
(e) Find the probability that R is more than twice the area of a square whose side has the length of the piece of string AP.

(4)

a) $X \sim U[0,9]$ Continuous Uniform distribution.

b) $P(X>6) = \frac{3}{9} = \frac{1}{3}$

c)



$A = x(9-x) = -x^2+9x$

$a=-1 \quad b=9$

d) $E(X) = \frac{9+0}{2} = 4.5$

$E(X^2) = \int x^2 f(x)\,dx = \frac{1}{9}\int_0^9 x^2\,dx = \frac{1}{27}[x^3]_0^9 = 27$

$\therefore E(R) = -1(27) + 9(4.5) = 13.5$

e) area of square $= x^2$

$P(x<3) = \frac{1}{3}$

$ax^2+bx > 2x^2$

$3x^2-9x<0$

$3x(x-3)<0 \quad \therefore x<3$